Business PreCalculus MATH 1643 Section 004, Spring 2014

Lesson 7: Quadratic Equations

Definition 1. **Quadratic Equation:** *A quadratic equation in the variable x is an equation of the form*

$$ax^2 + bx + c = 0$$

where a, b, and, c are real numbers and $a \neq 0$. Note that this form is called the standardform.

Example 1. $5x^2 - 2x + 3 = 0$ *is a quadratic equation with $a = 5$, $b = -2$, and, $c = 3$.*

Definition 2. **Zero-Product Property:** *Let A and B be two algebraic expressions. Then $AB = 0$ if and only if $A = 0$ or $B = 0$. For example, if $(x - 1)(x + 2) = 0$, then $x - 1 = 0$ or $x + 2 = 0$. Hence, $x = 1$ or $x = -2$.*

Definition 3. **Solving Quadratic Equation by Factoring:**

Step 1. *Write the given equation in the standard form.*

Step 2. *Factor the nonzero side of the equation from Step 1.*

Step 3. *Set each factor obtained from Step 2 equal to* 0.

Step 4. *Solve the resulting equations in Step 3.*

Step 5. *Check the solutions obtained in Step 4 in the original equation.*

Example 2. *Solve by factoring:* $2x^2 + 5x = 3$
Solution:

$$\begin{aligned} 2x^2 + 5x &= 3 && (\textit{original equation}) \\ 2x^2 + 5x - 3 &= 0 && (\textit{standard equation}) \\ (2x - 1)(x + 3) &= 0 && (\textit{factoring nonzero side}), \end{aligned}$$

then either $2x - 1 = 0$ or $x + 3 = 0$. So, the solutions are $x = \frac{1}{2}$ or $x = -3$. Finally, we need to check that these are valid solutions by plugging them in the original equation.

Example 3. *Solve by factoring:* $x^2 + 16 = 8x$
Solution:

$$\begin{aligned} x^2 + 16 &= 8x && (\textit{original equation}) \\ x^2 + 16 - 8x &= 0 && \\ x^2 - 8x + 16 &= 0 && (\textit{standard equation}) \\ (x - 4)(x - 4) &= 0 && (\textit{factoring nonzero side}), \end{aligned}$$

then either $x - 4 = 0$ or $x - 4 = 0$. So, we get $x = 4$. Finally, plugging 4 in $x^2 + 16 = 8x$ yields $32 = 32$ which means that 4 is the only solution of $x^2 + 16 = 8x$.

Definition 4. **Square-Root Property:** *Suppose u is any algebraic expression and $d \geq 0$. If $u^2 = d$, then $u = \pm\sqrt{d}$.*
For example, if $(x+3)^2 = 5$, then $x + 3 = \pm\sqrt{5}$. Then $x = -3 \pm \sqrt{5}$ and the solution set is $\{-3 - \sqrt{5}, -3 + \sqrt{5}\}$.

Definition 5. **Quadratic Formula:** *The solutions of the quadratic equation in the standard form $ax^2 + bx + c = 0$ with $a \neq 0$ are given by the formula*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example 4. *Solve $3x^2 = 5x + 2$ by using the quadratic formula.*
Solution: *The standard form of $3x^2 = 5x + 2$ is $3x^2 - 5x - 2 = 0$. Then the coefficients are: $a = 3$, $b = -5$, and $c = -2$. And we have*

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ x &= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(-2)}}{2(3)} \\ x &= \frac{5 \pm \sqrt{25 + 24)}}{6} \\ x &= \frac{5 \pm \sqrt{4}9}{6} \\ x &= \frac{5 \pm 7}{6} \end{aligned}$$

Then the solution set is $\{\frac{5-7}{6} = \frac{-1}{3}, \frac{5+7}{6} = 2\}$.